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Exact ground and excited states of a t–J ladder doped with two holes

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Abstract. A two-chain ladder model is considered that is described by the strong-coupling t–t'–J–J' Hamiltonian. For the case of two holes moving in a background of antiferromagnetically interacting spins, exact, analytical results are derived for the ground-state energy and low-lying excitation spectrum. The ground state is a bound state of two holes with total spin $S = 0$. The charge excitation is gapless and the spin excitation has a gap. The corresponding wave functions are also exactly determined. The bound hole pair is found to have symmetry of the d-wave type. In the limit of strong rung coupling, the model maps onto an effective hard-core-boson model which exhibits dominant superconducting pairing correlations.

1. Introduction

In the last few years, ladder systems have been studied extensively [1–3]. Interacting electron systems in one dimension (1D) are fairly well understood. There are several rigorous results available for such systems [3]. Powerful techniques like the Bethe *ansatz* [4] and bosonization [5] have yielded much useful information about such systems. After the discovery of high-T_c cuprate superconductivity, 2D interacting electron systems acquired new significance due to the fact that the dominant electronic and magnetic properties of the cuprate systems are associated with the CuO_2 plane [6, 7]. There are, however, very few rigorous results available for 2D systems. Ladders, consisting of n-chains coupled by rungs, interpolate between 1D and 2D and their study is expected to be useful for a proper understanding of interacting many-body systems. The possibility of deriving rigorous results is also greater. A number of ladder systems have been discovered recently exhibiting a variety of interesting phenomena [1–3]. Physical insight obtained from the study of ladders is also expected to be relevant for high-T_c cuprate systems. The cuprates, in the spin-disordered phase, are doped spin liquids. Below optimal doping levels and well above the superconducting transition temperature T_c, there are experimental signatures of a spin gap (SG) [7] opening up. The 'gap' has been ascribed to preformed Cooper pairs of holes which lack the long-range phase coherence of the superconducting state. The Cooper pairs become phase coherent only below T_c giving rise to superconductivity. Dagotto *et al* [8] were the first to show that a two-chain ladder has a spin-liquid ground state and a SG in the excitation spectrum. On doping the system with holes, binding of holes in pairs is possible, giving rise to dominant superconducting (SC) pairing correlations. A few years later, a hole-doped two-chain ladder system $Sr_{0.4}Ca_{13.6}Cu_{24}O_{41.84}$ was discovered which exhibits superconductivity under pressure [9].

The relationship between the 'pseudo-'spin gap, pre-formed hole pairs and superconductivity is not well understood in the case of cuprate systems. For the ladder system, the SG is a real gap and the binding of holes leading to SC pairing correlations can be explicitly demonstrated. Resistivity measurements for the ladder compound $(\mathrm{Sr},\mathrm{Ca})_{14}\mathrm{Cu}_{24}\mathrm{O}_{41}$ show unusual temperature dependence as in the case of cuprates [10] highlighting further similarities between the two systems. Bose and Gayen [11–14] have constructed a two-chain t–J-type ladder model for which several exact, analytical results can be derived in the undoped as well as doped cases. For two holes, the possibility of binding of holes was suggested but the bound-state spectrum was not derived. In section 2 of this paper, we give a detailed derivation of the low-lying spin and charge excitation spectrum of the ladder model in the two-hole sector. We show that the ground state consists of a bound pair of holes. The spin excitation spectrum has a gap and the charge excitation is gapless. The two-hole wave functions are also computed. The two-hole bound-state wave function is shown to have modified d-wave symmetry. All these results are exact and analytic in nature. The dominance of SC pairing correlations in the ladder model is shown in an approximate, analytical manner.

2. The exact two-hole excitation spectrum

The two-chain ladder model consists of two chains, each described by a t–J Hamiltonian, coupled by t'–J' interactions between them (figure 1). The model is described by the t–t'–J–J' Hamiltonian:

$$H = -\sum_{i,j,\sigma} t_{ij}(1-n_{i-\sigma})C^{\dagger}_{i,\sigma}C_{j,\sigma}(1-n_{j-\sigma}) + \text{h.c.} + \sum_{\langle ij\rangle} J_{ij}\vec{S}_i\cdot\vec{S}_j$$
$$= H_t + H_{t'} + H_J + H_{J'}. \qquad (1)$$

The constraint that no site can be doubly occupied is implied in the model. The hopping integral t_{ij} has the value t for nearest-neighbour (NN) hopping within a chain and also for diagonal transfer between chains (solid lines in figure 1). The corresponding spin–spin interactions J_{ij} are of strength J. The spins have magnitude 1/2. The hopping integral across vertical links (broken lines) connecting two chains has the strength t'. The corresponding spin–spin interaction strength J_{ij} is J'. We assume t and t' to be positive. In the conventional two-chain spin ladder, the diagonal interaction and hopping terms are absent. The inclusion of the diagonal terms of the same strength as the intra-chain ones enables one to reduce the difficult N-body problem to an easily solvable few-body problem. The conventional spin-ladder model, in the absence of diagonal terms, constitutes a many-body problem for which no simplification occurs. The only exact results, which are available, are numerical results based on exact diagonalization of finite ladders [1, 2, 15].

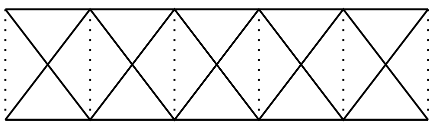

Figure 1. The spin-ladder model described by the t–t'–J–J' Hamiltonian (equation (1)).

In the half-filled limit, i.e., in the absence of holes, the t–t'–J–J' Hamiltonian in (1) reduces to $H_J + H_{J'}$. The exact ground state ψ_g (for $J' \geqslant 2J$) consists of singlets along the rungs of the ladder [11]. The ground-state energy $E_g = -(3J'/4)N$, where N is the number of rungs. An exact excited state can be constructed by replacing a singlet by a triplet. Creation of a triplet costs an amount of energy $J'/4$, so the spin gap $\Delta_{SG} = J'$. The excitation is localized and has no dynamics. Let us now consider the case of a single hole doped into the ladder. In the presence of holes a single rung can exist in nine possible states: (i) one empty state, (ii) two bonding states, (iii) two anti-bonding states, (iv) one singlet state and (v) three triplet states. These states are shown below:

$$\text{(i)}\ \begin{pmatrix} O \\ O \end{pmatrix}$$

$$\text{(ii)}\ \frac{1}{\sqrt{2}}\left(\begin{matrix}\uparrow\\ O\end{matrix} + \begin{matrix}O\\ \uparrow\end{matrix}\right), \frac{1}{\sqrt{2}}\left(\begin{matrix}\downarrow\\ O\end{matrix} + \begin{matrix}O\\ \downarrow\end{matrix}\right)$$

$$\text{(iii)}\ \frac{1}{\sqrt{2}}\left(\begin{matrix}\downarrow\\ O\end{matrix} - \begin{matrix}O\\ \downarrow\end{matrix}\right), \frac{1}{\sqrt{2}}\left(\begin{matrix}\uparrow\\ O\end{matrix} - \begin{matrix}O\\ \uparrow\end{matrix}\right)$$

$$\text{(iv)}\ \frac{1}{\sqrt{2}}\left(\begin{matrix}\uparrow\\ \downarrow\end{matrix} - \begin{matrix}\downarrow\\ \uparrow\end{matrix}\right)$$

$$\text{(v)}\ \begin{pmatrix}\uparrow\\ \uparrow\end{pmatrix}, \begin{pmatrix}\downarrow\\ \downarrow\end{pmatrix}, \frac{1}{\sqrt{2}}\left(\begin{matrix}\uparrow\\ \downarrow\end{matrix} + \begin{matrix}\downarrow\\ \uparrow\end{matrix}\right).$$

A single hole hops in a background of antiferromagnetically interacting spins. This, in general, is a difficult many-body problem because as the hole hops it gives rise to spin excitations in the system. The inclusion of diagonal hopping terms in our model leads to a cancellation of all the terms containing spin excitations, resulting in a perfect, coherent motion of the hole. We illustrate this through an explicit example. Consider a single hole in a bonding state, located in the mth rung. All other rungs are in singlet spin configurations. A pictorial representation of the state is

$$\Big|\Big|\Big|\cdots\frac{1}{\sqrt{2}}\left(\begin{matrix}\uparrow\\ O\end{matrix} + \begin{matrix}O\\ \uparrow\end{matrix}\right)_m\Big|_{(m+1)}\cdots \tag{2}$$

$$\frac{1}{\sqrt{2}}\left(\begin{matrix}\uparrow\\ O\end{matrix} + \begin{matrix}O\\ \uparrow\end{matrix}\right)_m\Big|_{(m+1)} \equiv \frac{1}{2}\left(\begin{matrix}\uparrow & \uparrow\\ O & \downarrow\end{matrix} - \begin{matrix}\uparrow & \downarrow\\ O & \uparrow\end{matrix} + \begin{matrix}O & \uparrow\\ \uparrow & \downarrow\end{matrix} - \begin{matrix}O & \downarrow\\ \uparrow & \uparrow\end{matrix}\right). \tag{3}$$

The state is an exact eigenstate of the J, J', t'-part of the t–t'–J–J' Hamiltonian. Let us now apply H_t to the state. Since background electrons are fermions, their ordering is important and one has to keep track of signs during interchanges. The ordering of fermions follows the convention

$$\begin{matrix}1\,3\,5\,\cdot\,\cdot\\ 2\,4\,6\,\cdot\,\cdot .\end{matrix}$$

On operating with H_t on the state shown in (3), one gets

$$H_t\begin{pmatrix}\uparrow & \uparrow\\ O & \downarrow\end{pmatrix} = t\begin{matrix}\uparrow & \uparrow\\ \downarrow & O\end{matrix} - t\begin{matrix}\uparrow & O\\ \uparrow & \downarrow\end{matrix}$$

$$-H_t\begin{pmatrix}\uparrow & \downarrow\\ O & \uparrow\end{pmatrix} = -t\begin{matrix}\uparrow & \downarrow\\ \uparrow & O\end{matrix} + t\begin{matrix}\uparrow & O\\ \downarrow & \uparrow\end{matrix}$$

$$H_t\begin{pmatrix}O & \uparrow\\ \uparrow & \downarrow\end{pmatrix} = t\begin{matrix}\uparrow & O\\ \uparrow & \downarrow\end{matrix} - t\begin{matrix}\downarrow & \uparrow\\ \uparrow & O\end{matrix}$$

$$-H_t\begin{pmatrix}O & \downarrow\\ \uparrow & \uparrow\end{pmatrix} = -t\begin{matrix}\downarrow & O\\ \uparrow & \uparrow\end{matrix} + t\begin{matrix}\uparrow & \downarrow\\ \uparrow & O\end{matrix} .$$

In each case, states in the second column are obtained due to diagonal hopping of the hole. There is a cancellation of the terms containing parallel spin pairs and the final state is given by

$$\Big|_m \frac{1}{\sqrt{2}}\begin{pmatrix} \uparrow & + & O \\ O & & \uparrow \end{pmatrix}_{m+1}. \tag{4}$$

One finds that the hole accompanied by a free spin-1/2 moves coherently by one lattice unit (compare with equation (3)). The eigenvalue problem now becomes very easy to solve. Let

$$\Psi(m) = \Big|\Big|\Big| \cdots \frac{1}{\sqrt{2}}\begin{pmatrix} \uparrow & + & O \\ O & & \uparrow \end{pmatrix}_m \Big|_{(m+1)} \cdots \tag{5}$$

$$\Psi = \frac{1}{\sqrt{N}} \sum_{m=1}^{N} e^{ikm}\Psi(m). \tag{6}$$

Ψ is an exact eigenstate of the t–t'–J–J' Hamiltonian with the energy eigenvalue

$$E_1 = 2t\cos(k) - t' + 3J'/4. \tag{7}$$

The energy is measured with respect to that of the ground-state energy in the undoped state. References [11,12] give a detailed discussion of the single-hole spectrum for both bonding and anti-bonding hole states. For conventional spin ladders, Troyer *et al* [15] have found numerical evidence of quasi-particle (QP) excitations carrying charge $+e$ and spin-1/2. The charge and spin may be located on different rungs. In the exact eigenstate of equation (6), the positively charged hole and the spin-1/2 are always located on the same rung. We refer to the composite object as a hole QP.

Let us now consider the case of two holes. The two holes can be introduced on the same rung or on separate rungs. Other rungs are in the singlet spin configurations. If the holes are located on two separate rungs, there are two free spins which can combine to give either a triplet or a singlet. In the triplet sector, the two hole QPs can scatter against each other giving rise to a continuum of scattering states with energy

$$E_{cont} = 4t\cos(K/2)\cos(q) - 2t' + 3J'/2. \tag{8}$$

K $(=k_1 + k_2)$ and q $(=(k_1 - k_2)/2)$ are the centre-of-mass momentum and the relative momentum wave vectors. The two-hole ground state belongs to the singlet sector. The exact eigenvalue equations have already been derived in reference [13] but a full analysis of these equations has so far not been carried out. Define the wave functions

$$\begin{aligned}\phi(m_1, m_2) = \frac{1}{2\sqrt{2}}\Big[\Big|\cdots\Big|\begin{pmatrix} \uparrow & + & O \\ O & & \uparrow \end{pmatrix}_{m_1}\Big|\cdots\begin{pmatrix} \downarrow & + & O \\ O & & \downarrow \end{pmatrix}_{m_2}\Big|\cdots\Big| \\ - \Big|\cdots\Big|\begin{pmatrix} \downarrow & + & O \\ O & & \downarrow \end{pmatrix}_{m_1}\Big|\cdots\begin{pmatrix} \uparrow & + & O \\ O & & \uparrow \end{pmatrix}_{m_2}\Big|\cdots\Big|\Big]\end{aligned} \tag{9}$$

and

$$\phi(m, m) = \Big|\Big|\cdots \begin{matrix} O \\ O_m \end{matrix} \cdots\Big|\Big|. \tag{10}$$

Define also the Fourier transforms

$$\phi(m, m + r) = \frac{1}{\sqrt{N}} \sum_K \exp[iK(m + r/2)]\phi_K(r) \tag{11}$$

for $0 \leqslant r \leqslant N/2 - 1$ and

$$\phi(m, m + N/2) = \sqrt{\frac{2}{N}} \sum_K \exp[iK(m + N/4)]\phi_K(N/2). \tag{12}$$

The two holes are separated by a distance r. From the periodic boundary condition and for $r \neq N/2$, the allowed values of K are $K = (2\pi/N)\lambda$, with $\lambda = 0, 1, 2, \ldots, N-1$. For $r = N/2$, the allowed values of K are odd multiples of $2\pi/N$. An eigenfunction in the momentum space is given by

$$\Psi_e^K = \sum_{r=0}^{N/2-1} a(r)\phi_K^r \tag{13}$$

where K is an even multiple of $2\pi/N$. When K is an odd multiple of $2\pi/N$, the eigenfunction is Ψ_0^K and the sum in equation (13) runs from 0 to $N/2$. The exact eigenvalue equations for both cases are given in reference [13]. When K is an even multiple of $2\pi/N$, the amplitudes $a(r)$ have the form

$$a(r) = \sin[q(N/2 - r)] \qquad \text{for } 1 \leqslant r \leqslant N/2 - 1. \tag{14}$$

The energy eigenvalues are obtained by simultaneously solving the equations

$$\epsilon = 2T\cos q \tag{15}$$

$$\epsilon + \frac{3J}{4} = \frac{4T^2}{\epsilon + 3J'/4 - 2t'} + \frac{T\sin[q(N/2-2)]}{\sin[q(N/2-1)]} \tag{16}$$

where $\epsilon = E - 3J'/2 + 2t'$ and, as before, energy E is measured w.r.t. that of the ground state in the undoped case. The energies for real values of q correspond to free hole states. Energies for bound and anti-bound states are obtained by making q complex. When T is +ve, making the changes $q \to \mathrm{i}q$ and $q \to \pi + \mathrm{i}q$, one gets the energies for anti-bound and bound states, respectively. When T is negative, the reverse is true. Similar results are obtained when K is an odd multiple of $2\pi/N$.

We now study the eigenvalue problem in the limit $N \to \infty$. The continuum of hole excited states, for real q, is given by equation (15). For complex q, bound and anti-bound states are obtained. Let us now replace q by $\pi + \mathrm{i}q$ in equations (15), (16). Since N is large, equation (16) reduces to

$$\epsilon + \frac{3J}{4} = \frac{4T^2}{\epsilon + 3J'/4 - 2t'} - T\mathrm{e}^{-q}. \tag{17}$$

From a simultaneous solution of equation (15) (with q replaced by $\pi + \mathrm{i}q$) and equation (17), one gets the following cubic equation in e^q:

$$\mathrm{e}^{3q} - \mathrm{e}^{2q}\left[\frac{3J + 3J'}{4T} - \frac{2t'}{T}\right] + \mathrm{e}^q\left[\frac{3J}{4T^2}(-2t' + 3J'/4) - 3\right] - \left(\frac{3J}{4T}\right) = 0. \tag{18}$$

The exact, analytic solutions of a cubic equation are given in reference [16]. For a physical solution, e^q is greater than or equal to 1. There are at most two physical solutions of the cubic equation in (18). Once a solution for e^q is obtained, the energy eigenvalue is obtained from equation (15) (with q replaced by $\pi + \mathrm{i}q$). For positive values of T, one gets the solution for a bound state of two holes and for T −ve, a solution for the anti-bound state is obtained. The other values of the excitation branches are obtained by symmetry. Figure 2 shows the exact energy spectrum for the bound state, a continuum of scattering states and anti-bound states of two holes for $J = t = t' = 1$ and $J' = 2J$. Figure 3 shows the same for the parameter values $J/t = 0.25$, $t = t' = 1$ and $J' = 2J$. The bound state of holes is obtained irrespective of the value of J/t being less than or greater than 1. Dagotto *et al* [8] were the first to show the binding of two holes in a two-chain ladder system. Their finding was based on exact diagonalization of finite-sized ladder systems. Later, Troyer *et al* [15] also found evidence for the binding of holes in finite ladder systems. In the case of our model, we have shown exactly

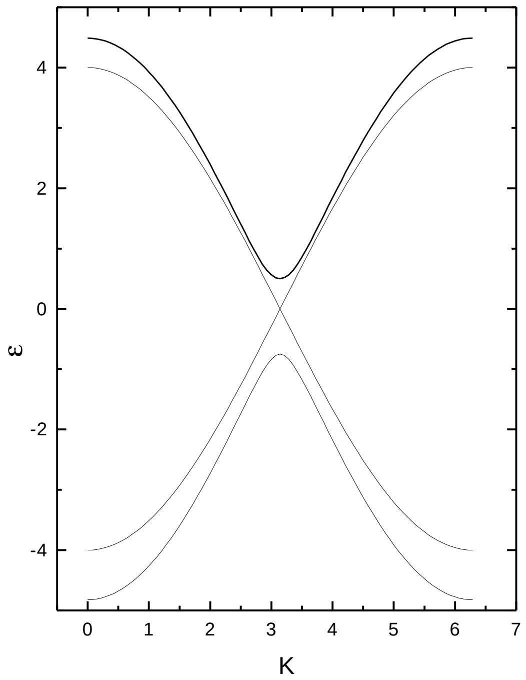

Figure 2. The exact energy spectrum (ϵ versus K) for the bound state, continuum of scattering states and anti-bound states of two holes ($J = t = t' = 1$, $J' = 2J$).

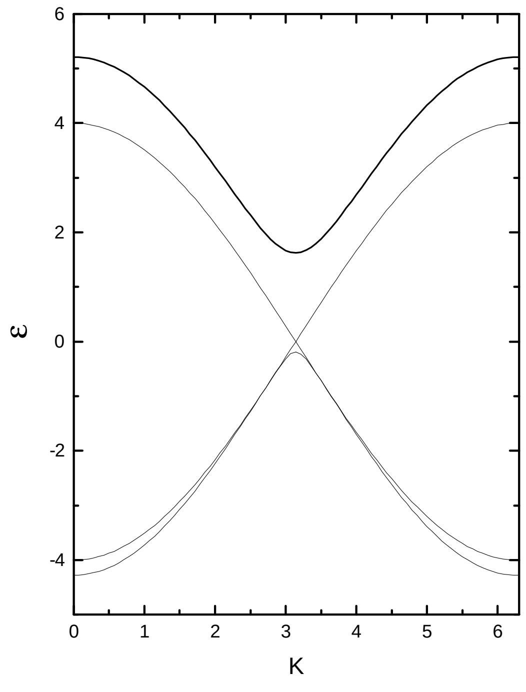

Figure 3. The exact energy spectrum (ϵ versus K) for the bound state, continuum of scattering states and anti-bound states of two holes ($J = 0.25$, $J' = 2J$, $t = t' = 1$).

and analytically the binding of two holes for $N \to \infty$. For finite systems also, one can solve the eigenvalue problem exactly.

The two-hole ground state is the bound state of two holes with centre-of-mass momentum wave vector $K = 0$. The exact bound-state wave function is given by (13) with $K = 0$ and q replaced by $\pi + \mathrm{i}q$ in (14). In the limit $N \to \infty$, one obtains

$$\frac{a(n)}{a(0)} = (-1)^{(n-1)} \mathrm{e}^{-(n-1)q} \frac{a(1)}{a(0)}. \tag{19}$$

This result shows explicitly that the bound-state wave function has an exponential decay as the separation between the two holes increases. With the knowledge of the eigenvalue ϵ, the ratio $a(1)/a(0)$ can be computed from the exact eigenvalue equations derived in reference [13]. Figure 4 shows a plot of $|a(r)/a(0)|^2$ versus r for the ground-state wave function with parameter values $J = t = t' = 1.0$ and $J' = 2J$ (dotted curve), $J' = 10J$ (solid curve). When J' is much larger than J, the holes prefer to be on the same rung to minimize the loss in exchange interaction energy. The hole delocalization energy along the rung is, however, lost. When J' and J are comparable, $|a(r)/a(0)|^2$ has maximum value when holes are separated by approximately one lattice constant. The exchange energy loss is less when two holes are on NN rungs than when they are further apart. Being on separate rungs, the holes gain in delocalization energy. The bound state is also more extended. These results are in agreement with the numerical results of Troyer *et al* [15].

The low-energy modes of a ladder system are characterized by their spin. Singlet and triplet excitations correspond to charge and spin modes respectively. The two-hole ground

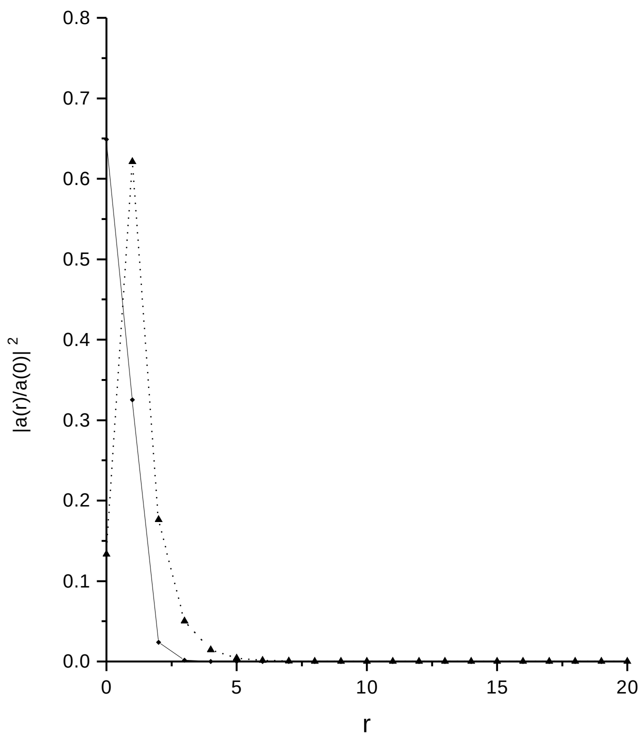

Figure 4. A plot of $|a(r)/a(0)|^2$ versus r for the ground-state wave function of two holes (equation (13)) ($J' = 2J$ (dotted curve), $J' = 10J$ (solid curve)).

state is in the singlet sector and, as already discussed, corresponds to a bound state of two holes for $K = 0$. Since a hole bound-state branch exists in the singlet sector, excitations with energy infinitesimally close to the ground-state energy are possible. These excitations are the charge excitations since the total spin is still zero and the charge excitation spectrum is gapless.

There are two distinct types of spin excitation. The first is the magnon ($S = 1$) excitation of the undoped ladder with energy J' measured with respect to the ground-state energy. The second type of spin-triplet excitation appears on doping the ladder. For a pair of holes, the lowest triplet excitation energy is $-4t - 2t' + 3J'/2$ from equation (8). The lowest triplet excitation energy depends on the values of t, t' and J'. The spin-gap energy Δ_{SG} is the difference in energies of the lowest triplet excitation and the ground-state (two-hole bound state in the singlet sector) energy. Figure 5 shows Δ_{SG} versus J/t for $t = t' = 1.0$ and $J' = 2J$. Thus, the two-chain ladder model has the feature that the charge excitation is gapless but the spin excitation has a gap. The same result holds true for the conventional spin ladder [2, 15]. In the notation CxSy [17] (x: gapless charge and y: gapless spin excitations), the t–J-type ladder model exists in the C1S0 (Luther–Emery) phase.

The experimental evidence of hole-based superconductivity [9] in a ladder system provides the motivation to look for superconducting pairing correlations in our ladder model. We have already shown the existence of the two-hole bound state. Define the pairing operator

$$\Delta_{ij} = c_{i\downarrow}c_{j\uparrow} - c_{i\uparrow}c_{j\downarrow} \tag{20}$$

and consider the quantity

$$\tilde{\Delta}_{ij} = \langle 2|\Delta_{ij}|0\rangle. \tag{21}$$

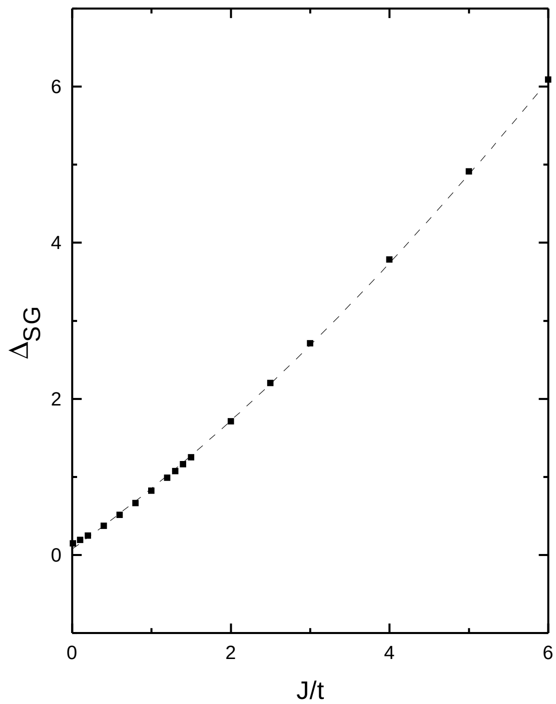

Figure 5. The spin gap Δ_{SG} versus J/t ($t = t' = 1.0$, $J' = 2J$).

$|0\rangle$ and $|2\rangle$ are the ground states in the case of zero and two holes respectively. For our ladder model, both of those ground states are exactly known and one can verify that $\tilde{\Delta}_{ii+\hat{x}}$ and $\tilde{\Delta}_{ii+\hat{y}}$ have opposite signs; $\hat{x}$ and $\hat{y}$ denote unit vectors in the x-direction (along the chain) and y-direction (along the rung). This is a signature of d-wave pairing and shows that the bound state of two holes has symmetry of the d-wave type. In the case of cuprate superconductors, there is much experimental evidence that the pairing wave function has d-wave symmetry [18].

In the large-J' limit, the ladder model can be mapped onto an effective boson model [15]. The physical picture is that of bound hole pairs existing along rungs and moving in a background of rung spin singlets. The hole pairs can be considered as hard-core bosons. The pair hopping matrix element to second order in perturbation theory is

$$t_b = \frac{2t^2}{3J'/4 - 2t'}. \tag{22}$$

There is also an interaction V_b between two hole pairs on NN rungs. To second order in perturbation theory,

$$V_b = \frac{4t^2}{3J'/4 - 2t'}. \tag{23}$$

Both t_b, $V_b \ll J'$ and one can map the ladder model onto an effective hard-core-boson model on a chain with NN interaction:

$$H_{eff} = -t_b \sum_i (b_i^\dagger b_{i+1} + \text{h.c.}) + V_b \sum_i n_i n_{i+1}. \tag{24}$$

$b_i^\dagger$ is the hard-core boson creation operator, creating a hole pair on the rung i and $n_i = b_i^\dagger b_i$ is the corresponding number operator. There is a well-known mapping between the effective boson model and the quantum XXZ spin model in a magnetic field [19], the Hamiltonian of which is given by

$$H_{xxz} = \sum_i [J_z S_i^z S_{i+1}^z + J_{xy}(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y)] - h \sum_i S_i^z. \tag{25}$$

The operator transformations connecting H_{eff} and H_{xxz} are

$$b_j = S_j^\dagger \qquad b_j^\dagger = S_j^- \qquad n_j = 1/2 - S_j^z. \tag{26}$$

There is a one-to-one correspondence between the phases of the spin model and those of the boson model. The disordered paramagnetic phase corresponds to the metallic phase for charged bosons. The AFM Néel-type order in the z-direction (when $J_z > J_{xy}$) describes the ordering of bosons on the lattice. For charged bosons, one obtains an insulating charge-ordered phase. The transition from the paramagnetic to the AFM phase represents a metal–insulator transition. The AFM XY order ($J_{xy} > J_z$) is characterized by a two-component order parameter and in the bosonic language corresponds to the off-diagonal long-range order of a superfluid condensate. For charged bosons, this is the SC phase.

For the XXZ chain, the asymptotic forms of the correlation functions have been obtained by Luther and Peschel using bosonization theory [20]. For $|J_z/J_{xy}| \leqslant 1$, the expressions for the correlation functions in the limit of large x and zero magnetic field are

$$\langle S^z(x,t) S^z \rangle \sim \cos(2k_F x) x^{(-1/\theta)} \tag{27}$$

$$\langle S^\dagger(x,t) S^- \rangle + \langle S^-(x,t) S^\dagger \rangle \sim x^{-\theta} \tag{28}$$

where the exponent

$$\theta = \frac{1}{2} - \pi^{-1} \arcsin(J_z/J_{xy}). \tag{29}$$

For the equivalent bosonic model, the correlation functions corresponding to (27) and (28) are the charge-density-wave (CDW) correlation $\langle n_r n_0 \rangle$ and the superconducting (SC) correlation $\langle b_r^+ b_0 \rangle$. The SC correlations are dominant if $\theta < 1$. For our ladder model, $J_z = V_b$ and $J_{xy} = -2t_b = -V_b$, i.e., for large r the CDW and SC pairing correlations exist. The transformed Hamiltonian (equation (25)), however, contains a magnetic field term. In the presence of the magnetic field h ($h = V_b$), the spin chain with $|J_z/J_{xy}| = 1$ is in a spin-flop phase [21] which is equivalent to the SC phase in the bosonic theory. Thus for our ladder model, the SC pairing correlations are dominant for large J'.

3. Conclusions

We have considered a two-chain t–J ladder model for which several exact, analytical results can be derived for the case of two holes. Inclusion of the diagonal exchange and hopping terms enables us to reduce the original N-body ($N - 2$ spins and two holes) problem to an effective two-body problem which is easily solved. The ground state is a bound state of two holes with centre-of-mass momentum wave vector $K = 0$ and total spin $S = 0$. The bound-state wave function has modified d-wave symmetry. The charge excitation is gapless whereas the spin excitation has a gap. All of the results derived by us are in agreement with the numerical results for the conventional two-chain spin ladder. In the strong-coupling limit, our results are the only exact, analytical results for the lightly doped two-chain t–J ladder. For more than two holes, we have not been able, as yet, to calculate the ground state and low-lying excitation spectrum exactly and analytically.

Recently, in a remarkable paper [22], Lin, Balents and Fisher have studied weakly interacting electrons hopping on a two-chain ladder. Using bosonization and perturbative renormalization-group (RG) analysis, they have shown that at half-filling the model scales onto the Gross–Neveu (GN) model. The GN model happens to be integrable and has SO(8) symmetry. For repulsive interactions, the two-chain ladder exhibits a Mott insulating phase at half-filling with d-wave pairing correlations. The exact energies of all of the low-lying excited states can be calculated because of the integrability of the GN model. Lin *et al* further studied the effects of doping a small density of holes into the d Mott spin-liquid phase at half-filling. Again, for a pair of holes, the ladder system exists in a SG phase with hole binding in the ground state and gapless charge excitations. Scalapino, Zhang and Hanke [23] have considered the strong-coupling limit of a two-chain ladder model with local interactions designed to exhibit exact SO(5) symmetry. This model too has a SG phase with hole pairs in the ground state. Numerical calculations on the t–J [15] and Hubbard ladders [24] also show the existence of such a phase. Thus, the SG phase with bound hole pairs appears to be a universal feature of the two-chain ladder system irrespective of the strength of the coupling. This phase also exhibits superconducting pairing correlations. For ladder systems the existence of a SG is favourable for the binding of holes. As mentioned in the introduction, the existence of a 'pseudo-SG' in the cuprates is conjectured to be associated with pre-formed Cooper pairs of holes. This conjecture is supported by our rigorous demonstration that the ground state in the SG phase consists of a bound pair of holes.

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